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THE CHOICE OF CONSTITUTIVE RELATIONS FOR AN ICE COVER†

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The constitutive relations between the internal stresses and the deformation parameters of a sea ice cover, which are used in the AIDJEX elastoplastic model and Hibler's non-linearly viscous model, are investigated. It is shown that the structural instability of the ice cover with respect to plastic shear deformations is a consequence of the associated flow rule used in these models. The use of constitutive relations which violate the associated flow rule, but which are in good agreement with the physical properties of granular media, is suggested. An ice cover damage parameter and an empirical equation which describes the change in this parameter are introduced into the treatment. Energy relations are investigated. © 1999 Elsevier Science Ltd. All rights reserved.

1. PLASTICITY CONDITIONS IN MODELS OF DRIFTING ICES

The model developed in the course of the large-scale AIDJEX experiment [1] and Hibler's model [2] are the best known and widely used mathematical models of drifting sea ice. In the AIDJEX model, the ice cover is modelled by a two-dimensional elastoplastic medium and, in Hibler's model, by a medium with a non-linearly viscous rheology. It is assumed in both approaches that the permissible internal stresses in the ice cover are located within a closed flow curve which is described by the equation

$$F(\sigma_\mathrm{I}, \sigma_\mathrm{II}, p_*) = 0;\ \ \sigma_\mathrm{I} = (\sigma_1 + \sigma_2)/2,\ \ \sigma_\mathrm{II} = (\sigma_1 - \sigma_2)/2 \tag{1.1}$$

where $\sigma_{1,2}$ are the principal values of the internal stress tensor $\sigma_{\alpha\beta}$ and p_* is a parameter which characterizes the maximum possible compressive pressure and is functionally dependent on the deformation tensor.

It is assumed that curve (1.1) is symmetrical about the σ_I axis and lies entirely in the domain $\sigma_\mathrm{I} < 0$. This last condition follows from the assumption that the ice cover does not resist tensile deformations.

The deformation rate tensor is related to the stress tensor by the associated flow rule

$$e_\mathrm{I} = \lambda \frac{\partial F}{\partial \sigma_\mathrm{I}},\ \ e_\mathrm{II} = \lambda \frac{\partial F}{\partial \sigma_\mathrm{II}},\ \ \lambda \geqslant 0;\ \ e_\mathrm{I} = e_1 + e_2,\ \ e_\mathrm{II} = e_1 - e_2 \tag{1.2}$$

where $e_{1,2}$ are the principal values of the deformation rate tensor $e_{\alpha\beta}$.

Equations (1.2) are satisfied when the stresses lie on the curve (1.1) and $\lambda \geqslant 0$. If the stresses $\sigma_\mathrm{I,II}$ lie inside the flow curve, the stress tensor in the AIDJEX model $\sigma_{\alpha\beta}$ is related to the deformation tensor $\varepsilon_{\alpha\beta}$ by Hooke's law while, in Hibler's model, $\sigma_{\alpha\beta}$ is related to the deformation rate tensor $e_{\alpha\beta}$ by a generalized Newton's law. In the latter case, the geometrical meaning of relation (1.2) lies in the fact that the actual stresses are equal to the projections of the viscous stresses onto the flow curve in the case when the stresses $\sigma_\mathrm{I,\,II}$, calculated using the formulae for the generalized Newton's law, prove to be outside the flow curve [3].

The associated flow rule in the theory of ideal plasticity and in theories of strain hardening materials follows from Drucker's postulate and the conditions for the invariance of the elastic moduli of a medium accompanying plastic deformations [4, 5]. In this case, the work done by the stresses in elastic deformation over a closed contour in the space of the stresses is always equal to zero.

The plasticity conditions (1.1) are set up in such a way that the flow curve "expands", that is, there is strengthening, when the parameter p_* increases. In the case of a decrease in p_*, the flow curve contracts, which corresponds to softening. The parameter p_* is usually defined in the form of a functional of the ice over density distribution function over the thicknesses.

The density distribution function satisfies the kinetic equation [1] which, in the case under consideration, replaces the law of conservation of mass

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$$dg/dt + ge_{\rm I} = \partial(fg)/\partial h + \psi, \psi = D(\alpha_0(\theta) + \alpha_r(\theta)w(g))$$
$$D = \sqrt{e_{\rm I}^2 + e_{\rm II}^2}, \theta = \mathrm{arctg}(e_{\rm II}/e_{\rm I}) \tag{1.3}$$

The function describes the redistribution of the ice over thicknesses during ridging and the formation of pure water. The coefficients α_0 and α_r determine the fractions of plastic deformation corresponding to the production of pure water and ridging. The function $f(h)$ is equal to the rate of melting or freezing over of the ice cover. The redistribution function satisfies the normalization conditions, which follow from the law of conservation of the mass of the ice cover during ridging and a change in the area occupied by the ice cover on the ocean surface accompanying plastic deformation. In greater detail, a change in the open water area and the area occupied by the ridging ice occurs due to the divergence or convergence of the ice cover, which does not take part in ridging. It follows from the normalization conditions that $\alpha_0 - \alpha_r = e_{\rm I}/D$. The coefficients α_0 and α_r are chosen in such a way that $\alpha_0(0) = 1$, $\alpha_r(\pi) = 1$ and $\alpha_0 = 0$ when $\theta \in (3/4\pi, \pi)$ and $\alpha_r = 0$ when $\theta \in (0, \pi/4)$.

The point $\theta = \pi/2$ corresponds to $e_{\rm I} = 0$. It follows from (1.2) that, at this point, $\partial F/\partial\sigma_{\rm I} = 0$ and the deformation reduces to a pure shear. Since, when $\theta = \pi/2$, the condition $\alpha_0 \neq 0$ is satisfied, then, even in the case of pure shear, part of the ocean surface is ice-free. This process will continue until the concentration of the ice cover reaches a critical value for which p_* vanishes and the ice cover is converted into a disperse medium without internal stresses. Hence, within the framework of the approach being considered, the ice cover is a structurally unstable material with respect to shear deformations. This corollary of the associated flow rule (1.2) and Eq. (1.3) is not fully substantiated from a physical point of view.

When $\theta \in (0, \pi/2)$, it follows from (1.2) that any shear deformation is accompanied by a bulk expansion. This consequence of the associated flow rule has been known for a long time [6, 7]. Experiments show that, for many types of granular media, the use of an associated flow rule leads to excess bulk deformations. As a rule, bulk expansion only accompanies shear during the initial instants of the motion and then ceases. Such an expansion is associated with the microscopic structure of the shear of a granular medium in which the individual granules roll across each other.

The second reason for criticism of an associated flow rule as applied to granular media is related to the example when the flow curve (1.1) contains rectilinear segments emanating from the origin of the coordinate system in the plane of the stresses ($\sigma_{\rm I}$, $\sigma_{\rm II}$). In these segments, the flow condition reduces to the Coulomb–Mohr dry friction law in which the stresses necessary for shear are proportional to the compression

$$\sigma_{\rm II} = -k_f\sigma_{\rm I} \tag{1.4}$$

It follows from (1.2) and (1.4) that the power of the internal stress is equal to zero; $\sigma_{\rm I}e_{\rm I} + \sigma_{\rm II}e_{\rm II} = 0$. This consequence of the associated law also does not correspond to physical ideas concerning shears of a free-flowing granular medium. At the same time, there is a good experimental confirmation of the Coulomb–Mohr friction law.

2. THE CONSTITUTIVE RELATIONS

Due to the inadequacies of the existing models, which have been noted above, the construction of a theory of granular media, which does not make use of an associated flow rule, is proposed. In the plane case, the constitutive relations are derived from kinematic hypotheses, the physical meaning of which reduces to the representation of any shear deformation in the form of the sum of two simple shears along slip lines which are the characteristics of the equilibrium equations [7].

We then derive two systems of constitutive relations, which it is convenient to use when modelling the motion of an ice cover. The first system consists of an equation of state and the Prandtl–Reuss equations written for the deviators of the stress and deformation rate tensors [8]. The equation of state has the form

$$\sigma_{\rm I} = -\pi(g) \tag{2.1}$$

where $\pi(g)$ determines the functional dependence of the pressure in the ice cover on the density distribution function of the ice cover with respect to the thicknesses $g(h)$ at which irreversible bulk deformations commence. Equation (2.1) determines the pressures required for the ridging of the ice under compression and the tensile stresses at which break up of the ice cover occurs.

The Prandtl–Reuss equations have the form

$$\frac{Ds}{Dt} + \lambda s = \mu(\sigma_{\rm I})\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y}\right), \quad \frac{D\tau}{Dt} + \lambda\tau = \mu(\sigma_{\rm I})\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$
$$s = (\sigma_{xx} - \sigma_{yy})/2, \quad \tau = \sigma_{xy} \tag{2.2}$$

Here, D/Dt is a Jaumann derivative, x, y, t are the horizontal coordinates and the time and u_x, u_y are the projections of the drift velocity of the ice cover onto the x- and y-axes. The dependence of the shear modulus μ on $\sigma_{\rm I}$ is selected in such a way that the slip lines coincide with the characteristics of Eqs (2.2) in the plane of u_x, u_y.

It follows from (2.1) and (2.2) that bulk deformations and shear are only related by the plasticity condition and actually occur independently. An ice cover is therefore a structurally stable medium with respect to shear deformations. Certain inadequacies in constitutive relations (2.1) and (2.2) may be due to the assumption that the pressure is independent of the shear deformations.

A second type of constitutive relations has the form [9]

$$e_{\alpha\beta} = \lambda\frac{\partial H}{\partial \sigma_{\alpha\beta}} + M\frac{Dt_{\alpha\beta}}{Dt}, \quad \lambda \geqslant 0; \quad s_{\alpha\beta} = \sqrt{2}\sigma_{\rm II}t_{\alpha\beta} \tag{2.3}$$

where $s_{xx} = -s_{yy} = s$, $s_{xy} = \tau$ are the components of the stress deviator and $H(\sigma_{\rm I}, \sigma_{\rm II})$ is the plastic potential which, from plasticity condition (2.1), cannot be identical to the function F. Equation (2.3) only describes the relation between the stresses and the deformation rates when plasticity condition (1.1) is satisfied and when $\lambda \geqslant 0$.

Equations (2.3) are hyperbolic equations if the inequalities

$$-1 < \frac{\partial H}{\partial \sigma_{\rm I}}\left(\frac{\partial H}{\partial \sigma_{\rm II}}\right)^{-1}, \quad -1 < 2M < 1 \tag{2.4}$$

are satisfied.

The velocity characteristics match the slip lines when the following relations are satisfied

$$\sin\Gamma = \frac{\sin\phi - \sin\nu}{1 - \sin\phi\sin\nu}, \quad \sin\Gamma = \frac{\cos\phi\cos\nu}{1 - \sin\phi\sin\nu}$$
$$\sin\Gamma = -2M, \quad \sin\nu = \frac{\partial g}{\partial \sigma_{\rm I}}\left(\frac{\partial g}{\partial \sigma_{\rm II}}\right)^{-1}, \quad \sin\phi = \frac{\partial F}{\partial \sigma_{\rm I}} \tag{2.5}$$

It follows from Eqs (2.3) that, when plasticity condition (1.1) is satisfied, the pressure in the ice cover depends on the bulk and shear deformation rates and is independent of the deformation tensor.

3. THE DAMAGE LEVEL OF AN ICE COVER

It has been pointed out above that the hypothesis that the ice does not resist tensile forces is customarily accepted when modelling an ice cover. In our opinion, the ability of an ice cover to resist tension depends on its damage.

A typical example is fast ice close to a bank. Fast ice withstands significant tensile loads when a wind blows off the land to the sea. If the wind speed exceeds a certain critical value, the fast ice can be fractured and become broken ice.

We will now define a parameter for the damageability of an ice cover, Σ, as the ratio of the total area of the ice floes found in a section of unit area of the ocean surface to the fraction of the area of this surface which is under the ice cover. For example, in the case of an unbounded, homogeneous ice plate $\Sigma = 2$, since the area of the ice surface is equal to the sum of the individual areas of the lower and upper surfaces of the ice cover. If there are cracks in the ice cover or it consists of separate ice floes or lumps of ice, then $\Sigma > 2$. Together with Σ, we define a parameter $\Sigma_d = A\Sigma$ which is equal to the total area of the ice floes in a region of unit area of the ocean surface.

It is assumed that the ice cover does not resist tension provided that its damage level is sufficiently high. For a low damage level, the ice cover resists tension and, when $\Sigma = 2$, the limit tensile stress is equal to the tensile strength, that is, it is equal to $h\sigma_{cr}$, where $\sigma_{cr} \approx 10^6 H/m^2$. An increase in damage is mainly due to processes involving irreversible plastic deformations of the ice cover under the action of external loads acting from the atmosphere and the ocean [11].

It is assumed that the ice cover possesses the ability to heal internal defects, provided that they are not formed at a very intense rate. The healing of damage is due to the freezing together of the edges of cracks and surfaces of ice floes when they come into contact with one another.

The ability of natural ice to heal internal defects has been investigated in many experimental papers (for example, [12]) and is due to the high homologous temperature of ice under natural conditions. The high intensity of healing of defects in fresh water ice is due to the presence of a quasi-fluid layer on its surface. In saline sea ice the intensity of healing is higher still, since the sea water, which permeates into cracks along numerous pores, rapidly fills them and then freezes.

We shall describe the process by which there is a change in the damage of a Lagrangian element of an ice cover by the empirical equation

$$\gamma\left(\frac{d\Sigma_d}{dt} + \Sigma_d e_1\right) = R_1(T, A, h, \Sigma, e_{\alpha\beta}, \mu_j) + R_2(T, A, h, \Sigma, \mu_j)$$
$$R_1 \geq 0, \quad R_2 \leq 0 \tag{3.1}$$

The quantity $\gamma\Sigma_d$ is called the surface energy of the ice cover [13]. The constant γ is equal to the bonding force between two layers of molecules in unit area of the material and T is the temperature of the ice cover.

The function R_1 determines the rate of increase in damage accompanying the deformation of the ice cover. It is assumed that the greatest increase in the damage as a consequence of the fracture of ice floes occurs during plastic deformation of the ice cover, which involves the displacement of the ice floes relative to one another. In this case, the rate of the damage accumulation depends on the rates of plastic deformation, the actual damage level, temperature, thickness and the concentration of the ice cover.

The function R_1 may also depend on other parameters, which characterize the state of the ice cover and the external action on it and are denoted by the letters μ_j.

A typical example is the effect of surface churning on the fracture of large ice fields, which manifests itself most strongly in the zone close to the edge. In this case, the parameter R_2 characterizes the amplitude and length of the waves which propagate under the ice. If the amplitude of the waves is sufficiently large, breakdown of the large-scale ice fields into ice floes occurs. The size of these ice floes is comparable with the wavelength.

The function R_2 determines the rate of healing of the damage. In order to investigate its form, an additional analysis if the experimental data on the thermodynamic properties of ice cover is required. We note that not only is the function R_2 unknown but the dependences of the rheological constants of the model on Σ are also unknown. The effect of these relations can be taken into account by writing down empirical equations of the form (3.1). For example, the change in the field force in the Hibler and AIDJEX models can be described by the empirical equation

$$\frac{dp_*}{dt} = R_1^p(T, A, h, \Sigma, e_{\alpha\beta}, \mu_j) + R_2^p(T, A, h, \Sigma, \mu_j), \quad R_1^p \geq 0, \quad R_2^p \leq 0 \tag{3.2}$$

The functions R_1^p and R_2^p are chosen empirically on the basis of intuitive ideas concerning the properties of an ice cover and comparison of the results of numerical calculations with natural observations. This involves an adjustment of the model for the geographical region under consideration.

4. ENERGY RELATIONS

We now consider energy relations for an ice cover, the internal structure of which is characterized by the thickness h, a concentration A, a damage Σ and a temperature T. We take the following expression for the surface density of the internal energy

$$U = \rho A(hu + \Delta g h^2), \Delta = (\rho_w - \rho)/\rho \tag{4.1}$$

where ρ_w is the density of water, ρ is the density of ice, and u is the bulk internal energy density. The second term is the potential energy of the ice cover as a body in a hydrostatic equilibrium [14].

We write the first and second laws of thermodynamics for the ice cover per unit area of the ocean surface as

$$dU = \sigma^{\alpha\beta} d\varepsilon_{\alpha\beta} + dQ^e + d_m U + d_T U$$

$$TdS = dQ^e + \tau_d^{\alpha\beta} d\varepsilon^p_{\alpha\beta} + dQ' + T(d_m S + d_T S) \tag{4.2}$$

$$(d_m U = -d\varepsilon^\alpha_\alpha U,\ d_m S = -d\varepsilon^\alpha_\alpha S,\ d_T U = \rho A u f dt,\ d_T S = \rho A s f dt)$$

Here, $S = \rho A h s$ and s are the surface and volume entropy density, aQ^e is the influx of heat and dQ' is the uncompensated heat. The total deformation tensor $\varepsilon_{\alpha\beta} = \varepsilon^e_{\alpha\beta} + \varepsilon^p_{\alpha\beta}$ is equal to the sum of the elastic and plastic deformation tensors, $d_m U$ and $d_m S$ are the changes in internal energy and entropy due to compressive or tensile deformations, $d_T U$ and $d_T S$ are the changes in the internal energy and entropy due to the thawing and freezing of the ice, and f is the rate of thawing or freezing of the ice.

We now determine the work done by the internal stresses during plastic deformation of the ice cover

$$\sigma^{\alpha\beta} d\varepsilon^p_{\alpha\beta} = (\tau_\Sigma^{\alpha\beta} + \tau_\psi^{\alpha\beta} + \tau_d^{\alpha\beta}) d\varepsilon^p_{\alpha\beta} \geqslant 0 \tag{4.3}$$

Here, $\tau_\Sigma^{\alpha\beta} d\varepsilon^p_{\alpha\beta} \geqslant 0$ is the part of the work of the internal stresses which is expanded in increasing the damage, $\tau_\psi^{\alpha\beta} d\varepsilon^p_{\alpha\beta} \geqslant 0$ is the part of the work of the stresses which is expended in the redistribution of lumps of ice during ridging or in the case of deformations which lead to a change in concentration, and $\tau_d^{\alpha\beta} d\varepsilon^p_{\alpha\beta} \geqslant 0$ is the part of the work done by the stresses which is converted into heat.

We now consider the free energy density function $F = u - Ts$ and assume that F depends on the temperature T, the elastic deformation tensor $\varepsilon^e_{\alpha\beta}$ and the damage of the ice averaged over the thickness Σ/h. Using formula (4.2) and taking account of the equation for the change in damage (3.1) and the law of conservation of mass

$$d(\rho A h) = -d\varepsilon^\alpha_\alpha \rho A h + \rho A f dt \tag{4.4}$$

we find

$$\frac{\partial F}{\partial T} = -s,\quad \rho A h \frac{\partial F}{\partial \varepsilon^e_{\alpha\beta}} = \sigma^{\alpha\beta},\quad \rho A h \frac{\partial F}{\partial \Sigma} R_1 = \tau_\Sigma^{\alpha\beta} d\varepsilon^p_{\alpha\beta}$$

$$\rho A h \frac{\partial F}{\partial \Sigma} R_2 = dQ',\quad \rho \Delta g h A dh = \tau_\psi^{\alpha\beta} d\varepsilon^p_{\alpha\beta} \tag{4.5}$$

It is clear that the specification of the free energy density function $F(T, \varepsilon^e_{\alpha\beta}, \Sigma/h)$ and the functions R_1, R_2 completely determines the dissipation in the medium. The sole condition imposed on these functions is that the expression for the dissipation $\tau_d^{\alpha\beta} d\varepsilon^p_{\alpha\beta}$ and the uncompensated heat dQ' must be positive.

The simplest expression for the free energy has the form

$$F = \frac{\varkappa_1}{2}(\varepsilon^e_{\rm I})^2 + \frac{\varkappa_2}{2}((\varepsilon^e_{\rm I})^2 + (\varepsilon^e_{\rm I})^2) - \alpha(T - T_0)\varepsilon^e_{\rm I} + \frac{\gamma}{h}\Sigma \tag{4.6}$$

where $\varkappa$ and $\varkappa_2$ are the elastic moduli and α is the coefficient of thermal expansion.

5. A MODEL OF BROKEN ICE

We shall refer to an ice cover which consists of small ice floes, which creep under one another when compressed and are barely fractured, as broken ice. The thawing and freezing of the ice are not considered. It is assumed that a broken ice cover is deformed without internal stresses until its concentration $A < 1$. The rheology of broken ice is therefore described by the relations $\sigma_{\alpha\beta} = 0$ when $A < 1$.

The damage of broken ice is quite large and u, its change, is described by the equation

$$d\Sigma_d + \Sigma_d d\varepsilon^\alpha_\alpha = 0 \tag{5.1}$$

We assume that ridging only occurs during compression of the ice, the concentration of which has attained the value $A = 1$. In this case, the change in the ice thickness is described by the equation

$$dh + hd\varepsilon_{\alpha}^{p,\alpha} = 0 \tag{5.2}$$

Note that the ridging scenario, described by Eq. (5.2), is not the same as the scenarios described in the AIDJEX models, where it is assumed that the thickness of the ice in a ridge is 4-5 times greater than the thickness of the ice from which the ridge is formed and, in this case, only a small part of the ice cover participates in ridging [1]. Formula (5.1) implies that an ice cover which forms a ridge possesses the properties of a granular material [15].

It follows from (4.5) that

$$\rho\Delta g h dh = \tau_{\psi}^{\alpha\beta} d\varepsilon_{\alpha\beta}^{p} \tag{5.3}$$

From (5.2) and (5.3) it follows that

$$\tau_{\psi}^{\alpha\beta} = -\delta^{\alpha\beta}\pi_{\psi}, \quad \pi_{\psi} = \rho\Delta g h^2/2 \tag{5.4}$$

We assume that the dissipation is the part of the work which is necessary to increase the potential energy during ridging

$$\tau_{d}^{\alpha\beta} d\varepsilon_{\alpha\beta}^{p} = k_r \tau_{\psi}^{\alpha\beta} d\varepsilon_{\alpha\beta}^{p} \tag{5.5}$$

It has been shown [15] that the dissipation may exceed the increase in the potential energy during ridging by a factor of 5–15 depending on the air temperature. Hence, $k_r \in (5.15)$ [14].

It follows from (5.4) and (5.5) that the condition for ridging has the form

$$\sigma_1 = -\pi_r, \quad \pi_r = (1 + k_r)\rho\Delta g h^2/2$$

We shall take the condition for the realization of plastic shear in the form of the Coulomb–Mohr law. The coefficient $k_f \in (0.1, 0.4)$ [14]. The Prandtl–Reuss equations (2.2) can be chosen as the constitutive relations describing shear deformations.

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REFERENCES

1. COON, M. D., MAYKUT, G. A., PRITCHARD, R. S., ROTHROCK, D. A. and THORNDIKE, A. S., Modeling the pack ice as an elastic-plastic material. *AIDJEX Bull.* No. 24, 1975, 1–105.
2. HIBLER, W. D. III, A dynamic thermodynamic sea ice model. *J. Phys. Oceanogr.*, 1979, **9**, 44, 815–846.
3. KLEINE, E. and SKLYAR, S., Mathematical features of Hibler's model of largescale sea-ice dynamics. *German J. Hydrography*, 1995, **47**, 179–230.
4. IL'YUSHIN, A. A., The growth of plastic deformation and yield surface. *Prikl. Mat. Mekh.*, 1960, **24**, 663–666.
5. RABOTNOV, Yu. N., *Mechanics of a Deformable Solid*. Nauka, Moscow, 1988.
6. MANDL, G. and LUQUE, R. F., Fully developed plastic shear flow of granular materials. *Geotechnique*, 1970, **20**, 277–307.
7. SPENCER, A. J., Deformation of ideal granular materials. *Mechanics of Solids: The Rodney Hill Anniversary Volume* (Edited by H. G. Hopkins and M. J. Sewell). Pergamon Press, Oxford, 1981, 607–652.
8. MARCHENKO, A. V., A model of a drifting ice cover. *Prikl. Mat. Mekh.*, 1994, **58**, 40–54.
9. HARRIS, D., Constitutive equations for planar deformations of rigid plastic materials. *J. Mech. Phys. Solids*, 1993, **41**, 1515–1531.
10. BOGORODSKII, V. V. and GAVRILO, V. P., *Ice. Physical Properties. Modern Methods of Glaciology*. Gidrometeoizdat, Leningrad, 1980.
11. SUKHORUKOV, K. K., The mechanism of the formation of the stressed state of sea ice accompanying macrofractures. *Dokl. Ross. Akad. Nauk*, 1996, **350**, 253–255.
12. MAENO, N., *The Science of Ice*. Mir, Moscow, 1988.
13. MELLOR, M., Mechanics behavior of sea ice. In: *The Geophysics of Sea Ice* (Edited by N. Untersteiner). Plenum Press, New York, 1986, 165–281.
14. ROTHROCK, D. A., The energetic of the plastic deformation of pack ice by ridging. *J. Geophys. Res.* 1975, **80**, 33, 4514–4519.
15. HOPKINS, M. A., HIBLER, W. D. III. and FLATO, G. M., On the numerical simulation of the sea ice ridging process. *J. Geophys. Res.*, 1991, **96**, C3, 4809–4820.

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